Effect of Proof Loading for High-Pressure Pipe Lines

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Introduction

Special pipelines in the processing plants of the oil industry require high strength steel manufactured by flow turning. Even with very careful manufacture and subsequent quality control one cannot exclude with certainty the existence of small cracks. Therefore, each pipe line segment will undergo a proof test with pressurized water before use. This test, however, is problematic. If small cracks of a size of less than 1 [mm] are present and which can hardly be detected by non-destructive testing a test at low pressures will not be very informative. At larger pressures around the mean operating pressure and higher a possibly present crack will grow a little and thus will reduce the reliability of the pipeline in further use. If cracks are not present the proof load test will increase the pipeline reliability significantly at higher proof load levels. At yet higher pressures the risk of failure during testing becomes larger. Any failure during proof loading, however, is likely to destroy the test setting due to the very high pressures involved.

Failure Model

For a circular interior crack with radius a one can use the following (simplified) instability criterion:

$$V = \left\{ \frac{L_r}{\left[8/\pi^2 \ln(\sec(\pi L_r/2)\right]^{1/2}} - K_r \le 0 \right\} \qquad \text{Note: } \sec(\varphi) = \frac{1}{\cos(\varphi)} \tag{1}$$

with

$K_r = \sigma\sqrt{\pi a}Y(a)/K_{1c}$ with a[m] (!) for K_{1c} in $\mathrm{MPa}\sqrt{\mathrm{m}}$

$\sigma = p_i r/t$

K_{1c} = fracture toughness

a = crack radius

$Y(a) \approx 1.12$ = geometry factor

$L_r = p_i/p_0$

p_i = internal pressure

$$p_0 = \left[1.07\frac{t}{r}\frac{(R_y+R_m)}{2}\right]\left[1-\frac{\pi a^2}{4t(a+t)}\right]$$

R_y = yield stress of steel

R_m = rupture strength of steel

t = wall thickness

r = cylinder radius

During proof loading the crack grows according to a crack growth relationship for ductile materials

$$\Delta a = \frac{\pi}{6(R_y K_{1c})^2}\frac{K_{1q}^4}{1-\left(\dfrac{K_{1q}}{K_{1c}}\right)^2} \tag{2}$$

with $K_{1q} = \sigma_q \sqrt{\pi a}$ the stress intensity factor at proof loading with pressure q producing stresses $\sigma_q = qr / t$. For failure during proof loading eq (1) holds but with

$$D = \left\{ \frac{L_r}{\left[8/\pi^2 \ln(\sec(\pi L_r / 2) \right]} - K_r \leq 0 \middle| \, p_i = q, a = a + \Delta a \right\} \tag{3}$$

The conditional failure probability is

$$P(V|\overline{D}) = \frac{P(V \cap \overline{D})}{P(\overline{D})} \tag{4}$$

where $\overline{D}$ is the complementary event to D.

For the numerical analysis the following assumptions are made:

Variable	Distribution function	Mean value	Coefficient of variation
p_i	Normal	10 [MPa]	0.10
R_y	Lognormal	1450 [MPa]	0.05
R_m	Normal	1700 [MPa]	0.05
K_{1c}	Lognormal	105 [MPa√m]	0.15
a	Rayleigh	1 [mm]	0.523
t	Normal	8.5 [mm]	0.02
r	Constant	600 [mm]	-

Non-zero correlations are $\rho(R_y, R_m) = \rho(R_y, K_{1c}) = \rho(R_m, K_{1c}) = -0.3$.

The reliability index for the event that v. Mises 2D-stress σ_v exceeds the yield strength R_y is $\beta_y = 9.3$ which is computed from the state function:

$$Y = \left\{ \sigma_v \geq R_y \right\} = \left\{ R_y - \frac{p_i r}{t} \frac{\sqrt{3}}{2} \leq 0 \right\} \tag{5}$$

Figure 1 shows the reliability index versus proof load level. The required reliability in terms of the reliability index, determined from overall system analysis, is β = 4.26. If no (or a very low) proof load is applied one sees that the required reliability level cannot be achieved. Also, for low proof load levels there is little effect on the safety level during operation. If, however, the proof load level is around the maximum expected operating pressure (MEOP = mean + 3σ = 13 [MPa]) the required reliability can be achieved. The reliability index increases significantly for larger proof load levels. At the same time the reliability index for the proof load test decreases. It means that at a proof load pressure of about MEOP, approximately two in one hundred of the proof load tests fail - a probably too large risk for the testing facility in mass production.

Note that the analysis is performed under the assumption that a crack exists and thus is still a conditional analysis. If it is assumed that there is only a small likelihood for cracks in each cylinder the reliability indices are much larger.

Cost-Benefit Analysis

The optimum proof load can be determined by cost optimization. The total cost are

$$E[C] = C_0 + C_q P(D) + HP(V \,|\, \bar{D}) \qquad (6)$$

where C_0 are the manufacturing cost, C_q the cost of failure during testing and H the failure cost during operation after survival of the proof load test. The individual and total cost function are drawn in figure 2 for $C_0 = 1$, $C_q = 100$ and $H = 5000$. In figure 2 it is seen that the optimal proof load is slightly below MEOP in this case and, of course, with the given cost factors. However, the problem is not very sensitive to small variations in the cost factors.

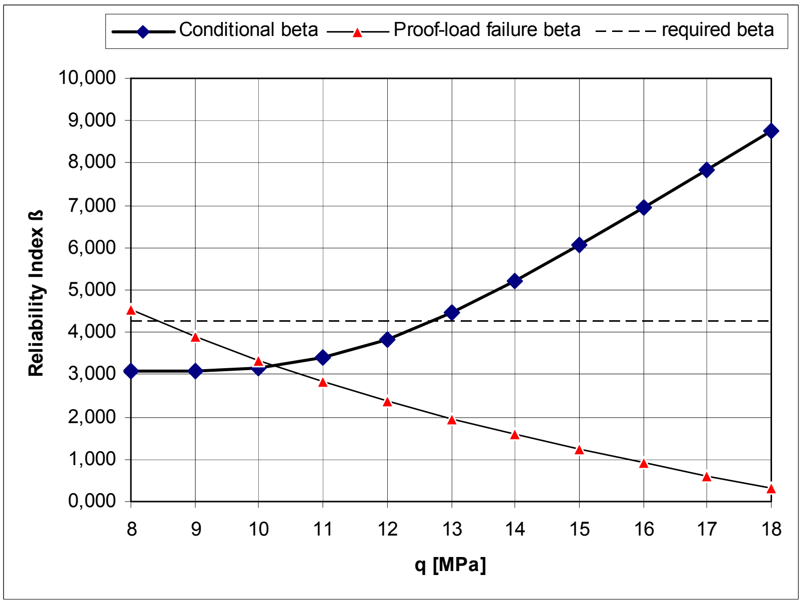

Figure 1: Reliability index for P(V|$\bar{D}$) and P(D)

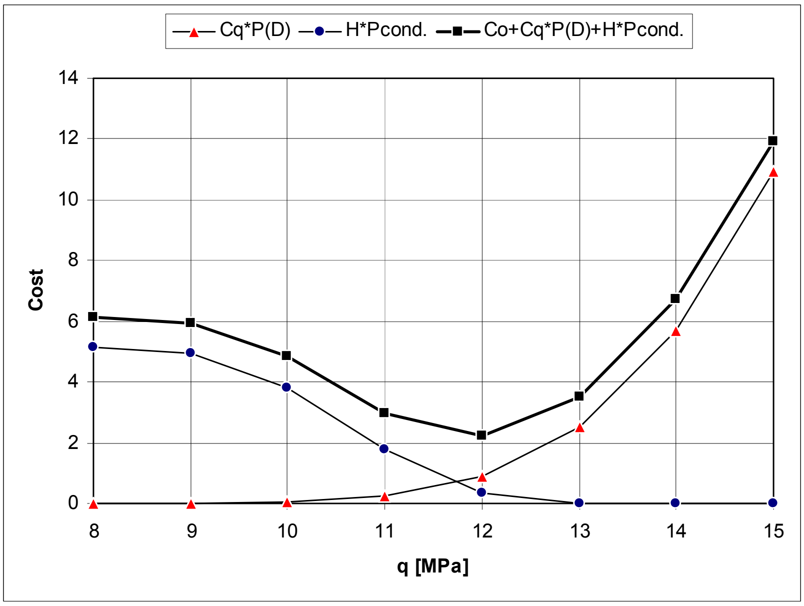

Figure 2: Cost functions

Gollwitzer, S.; Grimmelt, M.; Rackwitz, R: Brittle Fracture and Proof Loading of Metallic Pressure Vessels, Proc.: 7th Int. Conf. on Reliability and Maintainability, Brest, 1990